NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2013

09-04-2013 (Online-1)

IMPORTANT INSTRUCTIONS

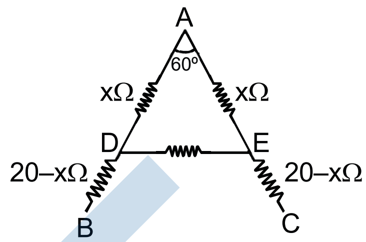
1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-PHYSICS

1. A letter 'A' is constructed of a uniform wire with resistance 1.0 Ω per cm. The sides of the letter are 20 cm and the cross piece in the middle is 10 cm long. The apex angle is 60°. The resistance between the ends of the legs is close to :

(A*) 26.7 Ω (B) 36.7 Ω (C) 50.0 Ω (D) 10 Ω

Sol. For ADE $\frac{1}{R'} = \frac{1}{2x} + \frac{1}{10}$



or $R' = \frac{20x}{10+2x}$

$R_{BC} = \frac{20x}{10+2x} + 20 - x + 20 - x$ ....(i)

or $\frac{20x}{10+2x} + 40 = 2x$

Solving we get

x = 10 Ω

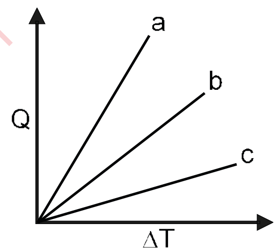
Putting the value of x = 10 Ω in equation (i)

We get

$R_{BC} = \frac{20 \times 10}{10 + 2 \times 10} + 20 = 10 + 20 - 10$

$= \frac{80}{3} = 26.7\Omega$

2. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a poly atomic (P) gas. The initial state of all the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a, b and c respectively correspond to :



(A) P, M and D (B) D, M and P (C*) M, D and P (D) P, D and M

Sol. On giving same amount of heat at constant pressure, there is no change in temperature for mono, dia and polyatomic.

$(\Delta Q)_P = \mu C_P \Delta T \left(\mu = \frac{\text{No. of molecules}}{\text{Avogadro's no.}} \right)$

or $\Delta T \propto \dfrac{1}{\text{no. of molecules}}$

3. From the following, the quantity (constructed from the basic constants of nature), that has the dimensions, as well as correct order of magnitude, vis-a-vis typical atomic size, is :

(A) $\dfrac{me^2}{4\pi \epsilon_0 h^2}$ (B*) $\dfrac{e^2}{4\pi \epsilon_0 mc^2}$ (C) $\dfrac{4\pi \epsilon_0 mc^2}{e^2}$ (D) $\dfrac{4\pi \epsilon_0 h^2}{me^2}$

Sol. As $E = mc^2$

also $E = F \cdot s = \dfrac{kq \cdot q}{r^2} \cdot r$

Therefore dimensionally

$$mc^2 = \frac{1}{4\pi \epsilon_0}\frac{q^2}{r}$$

$$\Rightarrow r = \frac{e^2}{4\pi \epsilon_0 mc^2}$$

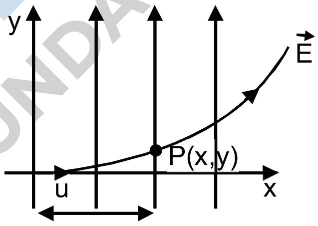
4. A uniform electric field $\vec{E}$ exists between the plates of a charged condenser. A charged particle enters the space between the plates and perpendicular to $\vec{E}$. The path of the particle between the plates is a :

(A) Hyperbola (B*) Parabola (C) Circle (D) Straight line

Sol. When charged particle enters perpendicularly in an electric field,

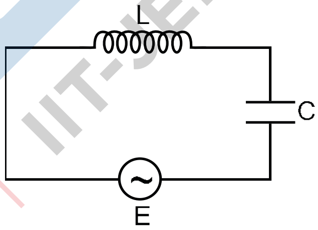
it describes a parabolic path

$$y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{4}\right)^2$$



This is the equation of parabola.

5. In the circuit shown here, the voltage across L and C are respectively 300 V and 400 V. The voltage E of the ac source is :



(A*) 100 Volt (B) 700 Volt (C) 500 Volt (D) 400 Volt

Sol. Voltage E of the ac source

$E = V_C - V_L$

$400\text{ V} - 300\text{ V} = 100\text{ V}$

6. When two sound waves travel in the same direction in a medium, the displacements of a particle located at 'x' at time 't' is given by :

$y_1 = 0.05 \cos (0.50\,\pi x - 100\,\pi t)$

$y_2 = 0.05 \cos (0.46\,\pi x - 92\,\pi t)$

where y_1, y_2 and x are in meters and t in seconds. The speed of sound in the medium is:

(A) 92 m/s (B) 332 m/s (C*) 200 m/s (D) 100 m/s

Sol. Standard equation

$$y(x,t) = A\cos\left(\frac{\omega}{V}x - \omega t\right)$$

From any of the displacement equation Say y_1

$$\frac{\omega}{V} = 0.50\pi \text{ and } \omega = 100\pi$$

$$\therefore \frac{100\pi}{V} = 0.5\pi$$

$$\therefore V = \frac{100\pi}{0.5\pi} = 200\,m/s$$

7. Two simple pendulums of length 1m and 4 m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed number of oscillations equal to:

(A) 7 (B) 3 (C*) 2 (D) 5

Sol. Let T_1 and T_2 be the time period of the two pendulums $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$ and $T_2 = 2\pi\sqrt{\frac{4}{g}}$

As $\ell_1 < \ell_2$ therefore $T_1 < T_2$

Let at t = 0 they start swinging together. Since their time periods are different, the swinging will not be in unison always. Only when number of completed oscillations differ by an integer, the two pendulums will again begin to swing together

Let longer length pendulum complete n oscillation and shorter length pendulum complete (n + 1) oscillation. For unison swinging

$(n + 1)T_1 = nT_2$

$$(n+1)\times 2\pi\sqrt{\frac{\ell}{g}} = (n)\times 2\pi\sqrt{\frac{4}{g}}$$

$\Rightarrow n = 1$

$\therefore$ N + 1 = 1 + 1 = 2

8. In the Bohr model an electron moves in a circular orbit around the proton. Considering the orbiting electron to b e a circular current loop, the magnetic moment of the hydrogen atom, when the electron is in nth excited state, is :

(A) $\left(\frac{e}{m}\right)\frac{n^2h}{2\pi}$ (B) $\left(\frac{e}{2m}\right)\frac{n^2h}{2\pi}$ (C*) $\left(\frac{e}{m}\right)\frac{nh}{2\pi}$ (D) $\left(\frac{e}{2m}\right)\frac{nh}{2\pi}$

Sol. Magnetic moment of the hydrogen atom, when the electron is in n^{th} excited state, i.e., n' = (n + 1)

As magnetic moment $M_n = I_nA = i_n\left(\pi r_n^2\right)$

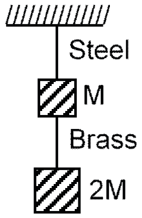
$$i_n = eV_n = \frac{mz^2e^5}{4\varepsilon_0^2n^3h^3}$$

$$r_n = \frac{n^2h^2}{4\pi^2kzmc^2}\left(k = \frac{1}{4\pi\in_0}\right)$$

Solving we get magnetic moment of the hydrogen atom for n^{th} excited state.

$$M_{n'} == \left(\frac{e}{2m}\right)\frac{nh}{2\pi}$$

9. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increases in their lengths is :



(A) $\frac{2a^2c}{b}$ (B*) $\frac{3a}{2b^2c}$ (C) $\frac{2ac}{b^2}$ (D) $\frac{3ac}{2ab^2}$

Sol. According to questions,

$$\frac{\ell_s}{\ell_b} = a, \frac{r_s}{r_b} = b, \frac{y_s}{y_b} = c, \frac{\Delta\ell_s}{\Delta\ell_b} = ?$$

As, $y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{F\ell}{Ay}$

$$\Delta\ell_s = \frac{3mg\ell_s}{\pi r_s^2 . y_s}[\because F_s = (M+2M)g]$$

$$\Delta\ell_b = \frac{2mg\ell_b}{\pi r_b^2 . y_b}[\because F_b = 2Mg]$$

$$\therefore \frac{\Delta\ell_s}{\Delta\ell_b} = \frac{\frac{3Mg\ell_s}{\pi r_s^2 . y_s}}{\frac{2Mg.\ell_b}{\pi r_b^2 . y_b}} = \frac{3a}{2b^3C}$$

10. On a linear temperature scale Y, water freezes at – 160° Y and boils at – 50°Y. On this Y scale, a temperature of 340 K would be read as : (water freezes at 273 K and boils at 373 K)

(A) – 233.7°Y (B) – 73.7°Y (C*) – 86.3°Y (D) – 106.3°Y

Sol. $\frac{\text{Reading on any scale} - \text{LFP}}{\text{UFP} - \text{LFP}}$

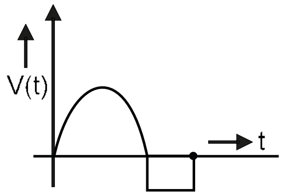
= constant for all scales

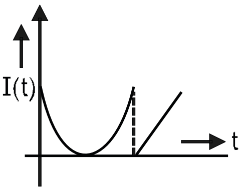
$$\frac{340-273}{373-273} = \frac{^\circ y - (-160)}{-50-(-160)}$$

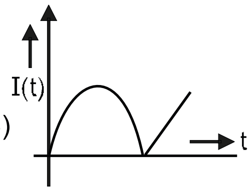
$$\Rightarrow \frac{67}{100} = \frac{y+160}{110}$$

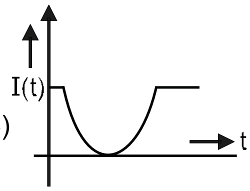
$\therefore$ y = – 86.3° y

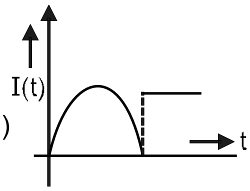
11. Two coils, X and Y, are kept in close vicinity of each other. When a varying current, I(t), flows through coil X, the induced emf V (t) in coil Y, varies in the manner shown here, The variation of I(t), with time, can then be represented by the graph labelled as graph :



(A*) 

(B) 

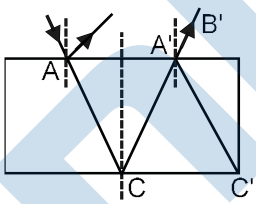
(C) 

(D) 

Sol. Induced emf

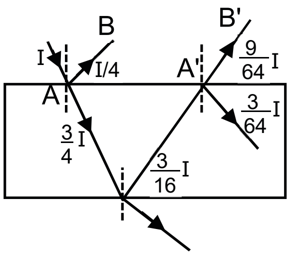
$\varepsilon \propto \frac{-di}{dt}$

12. A ray of light of intensity I is incident on a parallel glass slab at point A as shown in diagram. It undergoes partial reflection and refraction. At each reflection, 25% of incident energy is reflected. The rays AB and A' B' undergo interference. The ratio of I_{max} and I_{min} is :



(A) 4 : 1 (B*) 49 : 1 (C) 7 : 1 (D) 8 : 1

Sol. From figure $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{64}$



$\Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$

By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right)^2$

$= \left(\frac{\sqrt{\frac{9}{16}}+1}{\sqrt{\frac{9}{16}}-1}\right)^2 = \frac{49}{1}$

13. Two springs of force constants 300 N/m (Spring A) and 400 N/m (Spring B) are joined together in series. The combination is compressed by 8.75 cm. The ratio of energy stored in A and B is $\frac{E_A}{E_B}$. Then $\frac{E_A}{E_B}$ is equal to:

(A) $\frac{16}{9}$ (B) $\frac{9}{16}$ (C*) $\frac{4}{3}$ (D) $\frac{3}{4}$

Sol. Given : k_A = 300 N/m, k_B = 400 N/m

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B.

$x_B = (8.75 - x)$ cm

$F = 300 \times x = 400(8.75 - x)$

Solving we get, x = 5 cm

$x_B = 8.75 - 5 = 3.75$ cm

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300 \times (5)^2}{400 \times (3.75)^2} = \frac{4}{3}$$

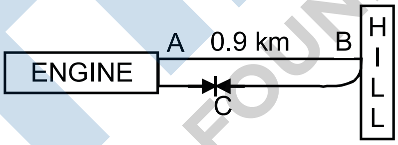
14. An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle whose echo is heard by the driver after 5 seconds. If the speed of sound in air is 330 m/s, then the speed of the engine is :

(A) 27.5 m/s (B*) 30 m/s (C) 32 m/s (D) 60 m/s

Sol. Let after 5 sec engine at point C

$$t = \frac{AB}{330} + \frac{BC}{330}$$

$$5 = \frac{0.9 \times 1000}{330} + \frac{BC}{330}$$



$\therefore$ BC 750 m

Distance travelled by engine in 5 sec

= 900 m – 750 m = 150 m

Therefore velocity of engine

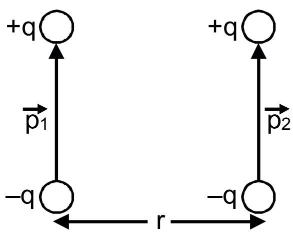
$$= \frac{150\text{m}}{5\,\text{sec}} = 30\,\text{m/s}$$

15. Two point dipoles of dipole moment $\vec{p}_1$ and $\vec{p}_2$ are at a distance x from each other and $\vec{p}_1 \parallel \vec{p}_2$. The force between the dipoles is :

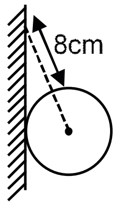
(A) $\frac{1}{4\pi\epsilon_0}\frac{6p_1p_2}{x^4}$ (B) $\frac{1}{4\pi\epsilon_0}\frac{8p_1p_2}{x^4}$ (C) $\frac{1}{4\pi\epsilon_0}\frac{4p_1p_2}{x^4}$ (D*) $\frac{1}{4\pi\epsilon_0}\frac{3p_1p_2}{x^3}$

Sol. Force of interaction

$$F = \frac{1}{4\pi\epsilon_0}\cdot\frac{3p_1p_2}{r^4}$$

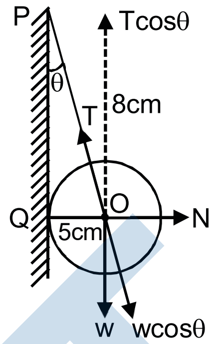


16. A uniform sphere of weight W and radius 5 cm is being held by a string as shown in the figure. The tension in the string will be :



(A*) $13\frac{W}{12}$ (B) $13\frac{W}{5}$

(C) $5\frac{W}{12}$ (D) $12\frac{W}{5}$



Sol. $PQ = \sqrt{OP^2 + OQ^2}$

$= \sqrt{13^2 + 5^2} = 12$

Tension in the string T, so $T\cos\theta = w$

$\Rightarrow T = \frac{w}{\cos\theta} = \frac{13}{12}W$

17. If a carrier wave $C(t) = A\sin\omega_c t$, were to be a amplitude modulated by a modulating signal $m(t) = A\sin w_m t$, the equation representing the modulated signal $[C_m(t)]$, and its modulation index, would be respectively :

(A) $C_m(t) = A(1 + \sin\omega_C t)\sin\omega_m t$ and 1 (B) $C_m(t) = A(1 + \sin\omega_m t)$ 1 and 2

(C*) $C_m(t) = A(1 + \sin\omega_m t)\sin\omega_C t$ and 1 (D) $C_m(t) = A(1 + \sin\omega_m t)\sin\omega_c t$ and 2

Sol. Modulation index

$m_a = \frac{E_m}{E_c} = \frac{A}{A} = 1$

Equation of modulated signal $[C_m(t)]$

$= E_{(C)} + m_a E_{(C)} \sin\omega_m t$

$= A(1 + \sin w_C t)\sin\omega_m t$

(As $E_{(C)} = A\sin\omega_C t$)

18. An electric current is flowing through a circular coil of radius R. The ratio of the magnetic field at the centre of the coil and that at a distance $2\sqrt{2}R$ from the centre of the coil and on its axis is :

(A) $2\sqrt{2}$ (B*) 27 (C) 36 (D) 8

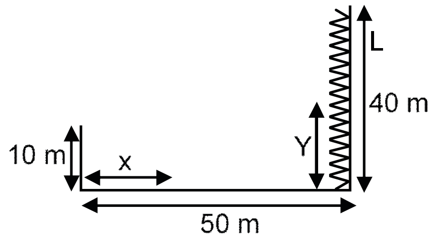
Sol. Given: Radius = R

Distance $x = 2\sqrt{2}R$

$$\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{R^2}\right) = \left(1 + \frac{(2\sqrt{2}R)^2}{R^2}\right)^{3/2}$$

$= (9)^{3/2} = 27$

19. A person lives in a high-rise building on the bank of a river 50 m wide. Across the river is a well lit tower of height 40 m. When the person, who is at a height of 10 m, looks through a polarizer at an appropriate angle at light of the tower reflecting from the river surface, he notes that intensity of light coming from distance X from his building is the least and this corresponds to the light coming from light bulbs at height 'Y' on the tower. The values of X and Y are respectively close to (refractive index of water 4/3)



(A*) 13 m, 27 m (B) 22 m, 13 m (C) 25 m, 10 m (D) 17 m, 20 m

20. Light is incident from a medium into air at two possible angles of incidence (a) 20° and (b) 40°. In the medium light travels 3.0 cm in 0.2 n. The ray will :

(A) Suffer total internal reflection in both cases (a) and (b)

(B) have 100% transmission in case (a)

(C) suffer total internal reflection in case (a)

(D*) suffer total internal reflection in case (b)

Sol. Velocity of light in medium

$$V_{mod} = \frac{3cm}{0.2ns} = \frac{3\times10^{-2}m}{0.2\times10^{-9}s} = 1.5m/s$$

Refractive index of the medium

$$\mu = \frac{V_{air}}{V_{med}} = \frac{3\times10^{8}}{1.5} = 2m/s$$

As $\mu = \frac{1}{\sin C}$

$$\therefore \quad \sin C = \frac{1}{\mu} = \frac{1}{2} = 30^{o}$$

Condition of TIR is angle of incidence I must be greater than critical angle. Hence ray will suffer TIR in case of (B) (i = 40º > 30º) only.

21. A shunt of resistance 1Ω is connected across a galvanometer of 120 Ω resistance. A current of 5.5 ampere gives full scale deflection in the galvanometer. The current that will give full scale deflection in the absence of the shunt is nearly:

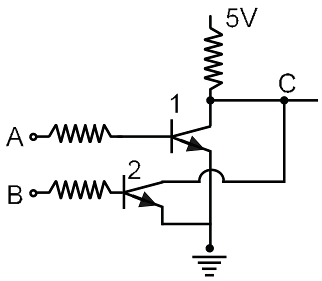
(A) 5.5 ampere (B) 0.5 ampere (C) 0.004 ampere (D*) 0.045 ampere

Sol. The current that will given full scale deflection in the absence of the shunt is nearly equal to the current through the galvanometer when shunt is connected i.e. I_g

As $I_g = \frac{IS}{G+S}$

$$= \frac{5.5\times1}{120+1} = 0.045 \text{ ampere.}$$

22. Consider two npn transistors as shown in figure. If 0 Volts corresponds to false and 5 volts correspond to true then the output at C corresponds to :



(A*) A NAND B (B) A OR B (C) A AND B (D) A NOR B

Sol. The output of C corresponds to A NAND

B or $\overline{A \cdot B} = C$

23. When Uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as:

${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3x + Q$ (energy) where three particles named x are produced and energy Q is released. What is the name of the particle x ?

(A) α-particle (B) electron (C*) neutron (D) neutrino

Sol. Nuclear fission equation

${}_{92}U^{135} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\ {}_{0}n^{1} + Q(\text{energy})$

Hence particle x is neutron.

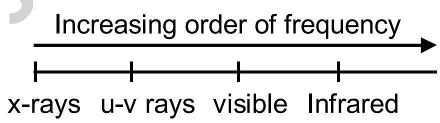
24. Photons of an electromagnetic radiation has an energy 11 keV each. To which region of electromagnetic spectrum does it belong ?

(A*) X-ray region (B) visible region (C) Ultra violet region (D) Infrared region

Sol. $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

$\Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{11 \times 1000 \times 1.6 \times 10^{-19}}$

= 12.4 Å



wavelength range of visible region is 4000Å to 7800Å.

25. Two balls of same mass and carrying equal charge are hung from a fixed support of length ℓ. At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, x between the balls is proportional to :

(A*) $\ell^{1/3}$ (B) ℓ^{2} (C) $\ell^{2/3}$ (D) ℓ

Sol. In equilibrium, $F_c = T \sin\theta$

$mg = T \cos\theta$

$\tan\theta = \frac{F_c}{mg} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$

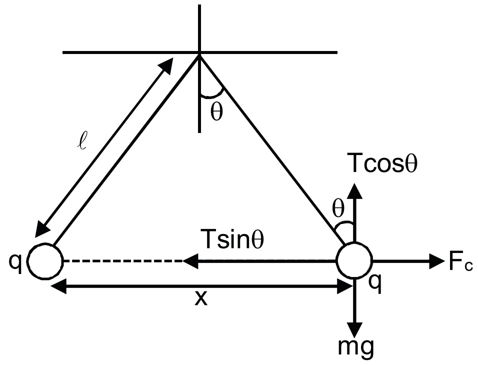
also $\tan\theta \approx \sin = \frac{x/2}{\ell}$

Hence, $\frac{x}{2\ell} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$

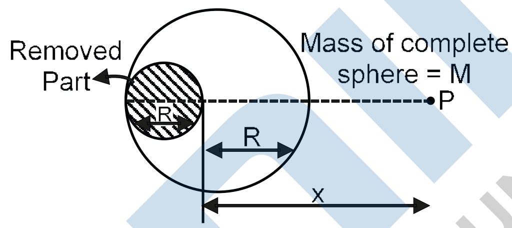
$\Rightarrow x^3 = \frac{2q^2\ell}{4\pi\epsilon_0 mg}$

$\therefore\ x = \left(\frac{q^2\ell}{2\pi\epsilon_0 mg}\right)^{1/3}$

Therefore $x \propto \ell^{1/3}$



26. The gravitational field, due to the 'left over part' of uniform sphere (from which a part as. shown, has been 'removed out '), at a very far off point, P, located as shown, would be (nearly):



(A) $\frac{8}{9}\frac{GM}{x^2}$ (B) $\frac{6}{7}\frac{GM}{x^2}$ (C) $\frac{5}{6}\frac{GM}{x^2}$ (D*) $\frac{7}{8}\frac{GM}{x^2}$

Sol. Let mass of smaller sphere (which has to be removed) is m,

Radius $= \frac{R}{2}$ (from figure)

$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$

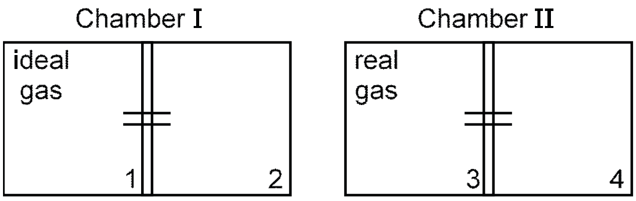
$\Rightarrow m = \frac{M}{8}$

Mass of the left over part of the sphere

$M' = M - \frac{M}{8} = \frac{7}{8}M$

Therefore gravitational field due to the left over part of the sphere

$= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$

27. 

There are two identical chambers, completely thermally insulated from surroundings. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and Compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum.

Statement 1 : No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature of real gas goes down (cooling) when it expands in vacuum.

Statement 2 : The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential

(A*) Statement 1 is false and statement 2 is true.

(B) Statement 1 and statement 2 both are true. But statement 2 is not the correct explanation of statement 1

(C) Statement 1 is true and Statement 2 is false.

(D) Statement 1 and Statement 2 both are true. Statement 2 is the correct explanation of Statement 1.

Sol. In ideal gases the molecules are considered as point particles and for point particle, there is no internal excitation, no vibration and no rotation. For an ideal gas the internal energy and for real gas both kinetics as well as potential energy.

28. In a metre bridge experiment null point is obtained at 40 cm from one end of the wire when resistance X is balanced against another resistance Y. If X < Y, then the new position of the null point from the same end, if one decides to balance a resistance of 3 X against Y, will be close to:

(A) 50 cm (B*) 67 cm (C) 75 cm (D) 80 cm

Ans. From equation, $\frac{x}{y} = \frac{40}{100-40} = \frac{2}{3}$

$\Rightarrow x = \frac{2}{3}y$

Again, $\frac{3x}{y} = \frac{Z}{100-Z}$

or $\frac{3 \times \frac{2y}{3}}{y} = \frac{Z}{100-Z}$

Solving we get Z = 67 cm

Therefore new position of null point

$\cong 67$ cm

29. This question has statement-1 and Statement-2. Of the four choices given after the statement, choose the one that best describes the two statements.

Statement 1 : A capillary is dipped in a liquid and liquid rises to a height h in it. As the temperature of the liquid is raised, the height h increases (if the density of the liquid and the angle of contact remain the same)

Statement 2 : Surface tension of a liquid decreases with the rise in its temperature.

(A) Statement-1 i true, Statement-2 is true, Statement-2 is the correct explanation for statement-1

(B) Statement-1 is true, Statement-2 is false

(C) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation for statement-1

(D*) Statement-1 is false, Statement-2 is true

Sol. Surface tension of a liquid decreases with the rise in temperature. At the boiling point of liquid, surface tension is zero.

Capillary rise $h = \frac{2T\cos\theta}{rdg}$

As surface tension T decreases with rise in temperature hence capillary rise also decreases.

30. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weights 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. The angular speed of the door just after the bullet embed into it will be :

(A*) 0.625 rad/sec (B) 6.25 rad/sec (C) 3.35 rad/sec (D) 0.335 rad/sec

PART-B-CHEMISTRY

31. In which of the following ionization processes the bond energy has increased and also the magnetic behaviour has changed from paramagnetic to diamagnetic ?

(A) $N_2 \longrightarrow N_2^+$ (B) $O_2 \longrightarrow O_2^+$ (C) $C_2 \longrightarrow C_2^+$ (D*) $NO \longrightarrow NO^+$

Sol. For NO

Total no. of electrons = 15

B.O = 2.5

Mag. Behaviour = Paramagnetic

For NO^+

Total no. of electrons 14

B.O = 3

Mag. Behaviour = Diamagnetic

32. Given :

$XNa_2HAsO_3 + Y\ NaBrO_3 + ZHCl \rightarrow NaBr + H_3AsO_4 + NaCl$

The values of X, Y and Z in the above redox reaction are respectively :

(A*) 3, 1, 6 (B) 2, 1, 2 (C) 2, 1, 3 (D) 3, 1, 4

Sol. On balancing the given reaction, we find $3Na_2HAsO_3 + NaBrO_3 + 6HCl \longrightarrow 6NaCl + 3H_3AsO_4 + NaBr$

33. An ether (A), $C_5H_{12}O$, when heated with excess of hot concentrated HI produced two alkyl halides which when treated with NaOH yielded compounds (B) and (C). Oxidation of (B) and (C) gave a propanone and an ethanoic acid respectively, The IUPAC name of the ether (A) is :

(A) ethoxypropane (B*) 2-ethoxypropane (C) 2-methoxybutane (D) methoxybutane

Sol. $\underset{(A)}{CH_3CH_2OCH(CH_3)_2} \xrightarrow{\text{Conc. HI}} \underset{(B)}{CH_3CH_2I} + \underset{(C)}{(CH_3)_2CHI}$

$CH_3CH_2I \xrightarrow{NaOH} CH_3CH_2OH \xrightarrow{[O]} CH_3COOH$

$(CH_3)_2CHI \xrightarrow{NaOH} (CH_3)_2CHOH \xrightarrow{[O]} (CH_3)_2CO$

Hence the IUPAC name of compound is

$CH_3 - CH_2 - O - \overset{2}{C}H(\,_3CH_3) - \overset{1}{C}H_3$

2-ethoxy propane

34. Which of the following compounds is not expected to show Lassaignes' test for nitrogen?

(A*) Hydroxylamine hydrochloride (B) Ethanamine

(C) Propanenitrile (D) Nitromethane

Sol. Lassaigne's test is used for the detection of nitrogen and given all nitrogenous compound except diazo $-(-N=N-)-$ compounds.

This test is shown only by the compounds containing C and N both hence hydroxyl amine hydrochloride ($H_2NOH.HCl$) will not perform this test.

35. Rate of dehydration of alcohol follows the order :

(A) 2° > 1° > CH_3OH > 3° (B) 2° > 3° > 1° > CH_3OH

(C) CH_3OH > 1° > 2° > 3° (D*) 3° > 2° > 1° > CH_3OH

Sol. The order of dehydration among three type of alcohols is 3° > 2° > 1° > CH_3OH. This behaviour is related to the relative stabilities o carbocations (3° > 2° > 1°).

36. Sodium carbonate cannot be used in place of $(NH_4)_2CO_3$ for the identification of Ca^{2+} , Ba^{2+} and Sr^{2+} ions (in group V) during mixture analysis because :

(A) Mg^{2+} ions will also be precipitated

(B) sodium ions will react with acid radicals

(C) Concentration of CO_3^{2-} ions is very low

(D*) Na^+ ions will interfere with the detection of Ca^{2+}, Ba^{2+}, Sr^{2+} ions

Sol. If Na_2CO_3 is used in place of $(NH_4)_2CO_3$. It will precipitate group V radicals as well as magnesium radicals. The reason for this is the high ionization of Na_2CO_3 in water into Na^+ and CO_3^{--}. Now the higher concentration of CO_3^{--} is available which exceeds the solubility product of group V radicals as well as that of magnesium radicals.

37. 12 g of a nonvolatile solute dissolved in 108 g of water produces the relative lowering of vapour pressure of 0.1. The molecular mass of the solute is :

(A) 60 (B) 40 (C*) 20 (D) 80

Sol. $$\frac{P^0 - P_s}{P^0} = \frac{n}{N} = \frac{w}{m} \times \frac{M}{W}$$

$$0.1 = \frac{12}{m} \times \frac{18}{108}$$

$$m = \frac{12\times}{0.1 \times 108} = 20$$

38. Potassium dichromate when heated with concentrated sulphuric acid and a soluble chloride, gives brown-red vapours of :

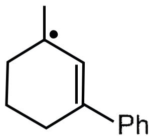
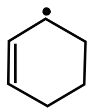
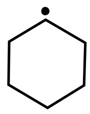
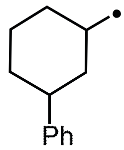
(A) CrO_3 (B*) CrO_2Cl_2 (C) Cr_2O_3 (D) $CrCl_3$

Sol. Solid potassium dichromate when heated with concentrated sulphuric acid and a soluble chloride gives orange red vapours o a volatile oily liquid CrO_2Cl_2

$K_2Cr_2O_7 + 4NaCl + 6H_2SO_4 \longrightarrow 2KHSO_4 + 4NaHSO_4 + 2CrO_2I_2$

chromyl chloride

39. Which one of the following is most Stable ?

(A*)  (B)  (C)  (D) 

Sol. 3° carbocations are most stable.

40. The instantaneous rate of disappearance of MnO_4^- ion in the following reaction is $4.56\times10^{-3} Ms^{-1}$

$2\,MnO_4^- + 10\,I^- + 16\,H^+ \rightarrow 2\,Mn^{2+} + 5\,I_2 + 8\,H_2O$

The rate of appearance I_2 is :

(A) 1.14×10^{-3} Ms^{-1} (B) 5.7×10^{-3} Ms^{-1} (C) 4.56×10^{-4} Ms^{-1} (D*) 1.14×10^{-2} Ms^{-1}

Sol. Given $-\frac{dMnO_4^-}{dt} = 4.56\times10^{-3} Ms^{-1}$

From the reaction given,

$$-\frac{1}{2}\frac{dMnO_4^-}{dt} = \frac{4.56\times10^{-3}}{2} Ms^{-1}$$

$$-\frac{1}{2}\frac{dMnO_4^-}{dt} = \frac{1}{5}\frac{dI_2}{dt}$$

$$\therefore\ -\frac{5}{2}\frac{dMnO_4^-}{dt} = \frac{dI_2}{dt}$$

On substituting the given value

$$\therefore\ -\frac{dI_2}{dt} = \frac{4.56\times10^{-3}\times5}{2} = 1.14\times10^{-2}\ M/s$$

41. The migration of dispersion medium under the influence of an electric potential is called :

(A) Cataphoresis (B*) Electrophoresis (C) Sedimentation (D) Electroosmosis

Sol. The motion of a liquid through a membrane under the influence of an applied electric field is known as electro-osmosis.

42. The addition of HI in the presence of peroxide catalyst does not follow anti Markovnikov's rule because:

(A) 1 atom combines with H atom to give back HI

(B*) Iodine atom is not reactive enough to add across a double bond

(C) HI is a strong reducing agent

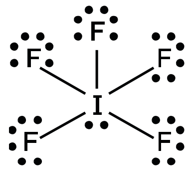
(D) H-I bond is too strong to be broken homolytically

Sol. HI does not exhibit peroxide effect. HI bond although dissociates easily into iodine radicals, they being bigger in size are not much reactive but recombine together to form iodine molecule.

43. Which one of the following molecules is polar?

(A) CF_4 (B*) IF_5 (C) XeF_4 (D) SbF_5

Sol. The geometry of IF_5 is square pyramidal with an unsymmetric charge distribution therefore this molecule is polar.



44. An element having an atomic radius of 0.14 nm crystallizes in an fcc unit cell. What is the length of a side of the cell ?

(A*) 0.4 nm (B) 0.96 nm (C) 0.56 (D) 0.24 nm

Sol. For a fcc unit cell

$$r = \frac{\sqrt{2}a}{4}$$

$$a = \frac{4r}{\sqrt{2}} = 2\sqrt{2} \times 0.14 = 0.39 \approx 0.4\text{nm}$$

45. Calcination is the process in which :

(A) ore is heated above its melting point to expel H_2O or CO_2 or SO_2

(B) ore is heated below its melting point to expel volatile impurities

(C) ore is hated above its melting point to remove S, As and Sb as SO_2, AS_2O_3 and Sb_2O_3 respectively

(D*) ore is heated below its melting point to expel H_2O or CO_2

Sol. Calcination is a process of heating a substance to a high temperature but below the melting or fusion point, causing loss of moisture, reduction or oxidation and dissociation into simpler substances.

46. Formaldehyde can be distinguished from acetaldehyde by the use of :

(A) Tollen's reagent (B*) I_2/Alkali (C) Fehling's solution (D) Schiff's reagent

Sol. Only acetaldehyde and methyl ketones give iodoform test.

47. Given :

(a) $H_2(g) + ½O_2(g) \rightarrow H_2O(\ell)$; $\Delta H^o_{298K} = -285.9$ kJ mol^{-1}

(b) $H_2(g) + ½O_2(g) \rightarrow H_2O(g)$; $\Delta H^o_{298K} = -241.8$ kJ mol^{-1}

The molar enthalpy of vapourisation of water will be

(A) 527.7 kJ mol^{-1} (B*) 44.1 kJ mol^{-1} (C) 241.8 kJ mol^{-1} (D) 22.0 kJ mol^{-1}

Sol. Given :

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell); \Delta H^o = -285.9\text{kJmol}^{-1} \quad ....(1)$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g); \Delta H^o = -241.8\text{kJmol}^{-1} \quad ....(2)$$

$H_2O(l) \longrightarrow H_2O(g)$; ΔH^o = ?

On subtracting equation. (2) from equation (1) we get

$H_2O(\ell) \longrightarrow H_2O(g)$; $\Delta H^o = -241.8 - (-285.9) = 44.1$ kJ mol^{-1}

48. Which of the following enzyme converts starch into maltose ?

(A) Maltase (B) Zymase (C) Invertase (D*) Diastase

Sol. Maltose is obtained by partial hydrolysis of starch by the enzyme diastase present in Malt.

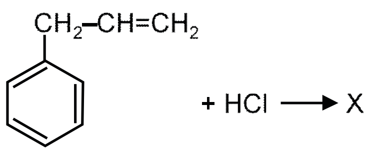
$$2(C_6H_{10}O_5)_n + nH_2O \xrightarrow{\text{Distance}} nC_6H_{12}O_6\text{(Maltose)}$$

49. Trigonal bipyramida geometry is shown by :

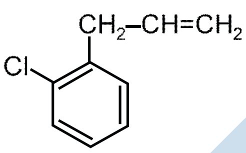
(A) $[XeF_8]^{2-}$ (B) $XeOF_2$ (C*) XeO_3F_2 (D) $FXeOSO_2F$

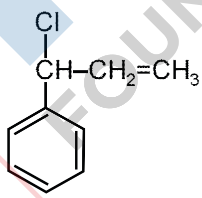
Sol. The hybridization o XeO_3F_2 is sp^3d and its structure is trigonal bipyramidal in which oxygen atoms are situated on the plane and the fluoride atoms are on the top and bottom.

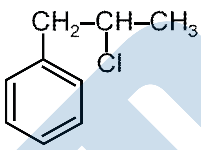
50. Given :

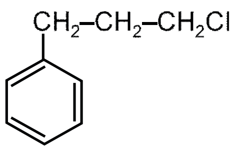


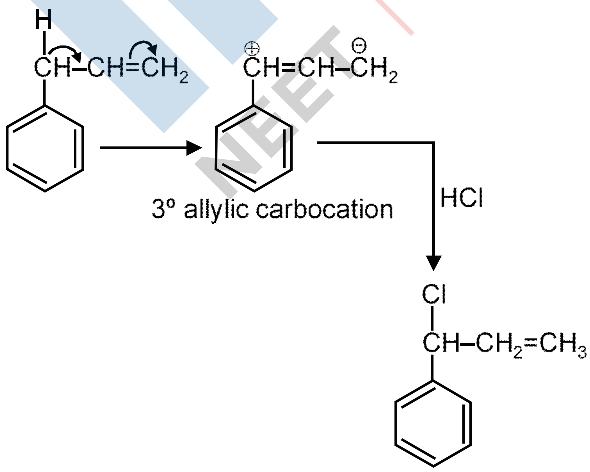
X is :

(A) 

(B*) 

(C) 

(D) 

Sol. 

(Since tertiary carbocation is more stable)

51. Electron gain enthalpy with negative sign of fluorine is less than that of chlorine due to :

(A) Bigger size of 2p orbital of fluorine
(B) High ionization enthalpy of fluorine
(C) Smaller size of chlorine atom
(D*) Smaller size of fluorine atom

Sol. The electron gain enthalpy order for halogens is

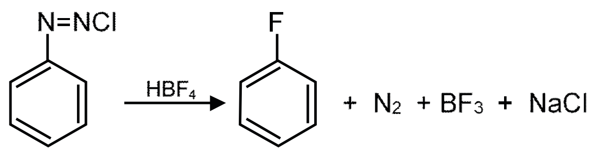
Cl > F > Br > I

Due to small size of fluorine the extra electron to be added feels more electron-electron repulsion. Therefore fluorine has less value for electron affinity than chlorine.

52. Aryl fluoride may be prepared from arene diazonium chloride using :

(A) HBF_4 / $NaNO_2$, Cu , Δ
(B) CuF / HF
(C) Cu / HF
(D*) HBF_4 / Δ

Sol.



(Balz-Schiemann's reaction)

53. Type of isomerism which exists between $[Pd(C_6H_5)_2(SCN)_2]$ and $[Pd(C_6H_5)_2(NCS)_2]$ is :

(A) Solvate isomerism
(B*) Linkage isomerism
(C) Ionisation isomerism
(D) Coordination isomerism

Sol. The compound shows linkage isomerism because the ligand in the compound is an ambidentate ligand that can bond at more than one atomic site.

i.e. $:NCS^-$ and $:SCN^-$

54. By how many folds the temperature of a gas would increase when the root mean square velocity of the gas molecules in a container of fixed volume is increased from 5×10^4 cm/s to 10×10^4 cm/s ?

(A*) Four
(B) Six
(C) Two
(D) Three

Sol. r.m.s. velocity $V_{rms} = \sqrt{\frac{3RT}{M}}$

i.e. $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

$\frac{5 \times 10^4}{10 \times 10^4} = \frac{1}{2} = \sqrt{\frac{T_1}{T_2}}$

$\therefore$ $T_2 = 4T_1$

55. In reaction $A + 2B \rightleftharpoons 2C + D$, initial concentration of B was 1.5 times of [A], but at equilibrium the concentrations of A and B became equal. The equilibrium constant for the reaction is :

(A) 12
(B) 8
(C*) 4
(D) 6

Sol.

A	+	2B	$\rightleftharpoons$	2C	+	D
a		1.5a		0		0

$(a - x) \quad (1.5a - 2x) \qquad 2x \qquad x$

Hence $K_C = \dfrac{(2x)^2 \times x}{(a-x)(1.5a-2x)^2}$

Given, at equilibrium

$\therefore \quad (a - x) = (1.5a - 2x)$

$\therefore \quad a = 2x$

On solving $K_C = 4$

56. The element with which of the following outer electron configuration may exhibit the largest number of oxidation states in its compounds :

(A) $3d^6 4s^2$ (B) $3d^8 4s^2$ (C) $3d^7 4s^2$ (D*) $3d^5 4s^2$

Sol. The element with outer electron configuration $3d^5 4s2$ is Mn which exhibits oxidation states from +2 to +7.

57. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At which concentration of Ba^{2+}, precipitate of $BaCO_3$ begins to form ? (K_{sp} for $BaCO_3 = 5.1 \times 10^{-9}$)

(A) 7.1×10^{-8} M (B) 8.1×10^{-7} M (C) 4.1×10^{-5} M (D*) 5.1×10^{-5} M

Sol. Given $Na_2CO_3 = 1.0 \times 10^{-4}$ M

$\therefore \quad [CO_3^{--}] = 1.0 \times 10^{-4}$ M

i.e. $s = 1.0 \times 10^{-4}$ M

At equilibrium

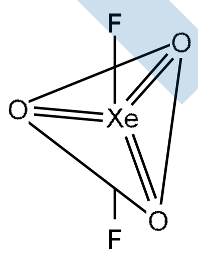
$[Ba^{++}][CO_3^{--}] = K_{sp}$ of $BaCO_3$

$$[Ba^{++}] = \frac{K_{sp}}{[CO_3^{--}]} = \frac{5.1 \times 10^{-9}}{1.0 \times 10^{-4}} = 5.1 \times 10^{-5} M$$

58. If a polythene sample contains two monodisperse fractions in the ratio 2 : 3 with degree of polymerization 100 and 200, respectively, then its weight average molecular weight will be :

(A) 4300 (B*) 4900 (C) 5200 (D) 4600

Sol.



59. Electrode potentials (E°) are given below :

$Cu^+ / Cu = + 0.52$ V,

$Fe^{3+} / Fe^{2+} = + 0.77$ V

$\frac{1}{2} I_2(s) / I^- = + 0.54$ V,

$Ag^{+} / Ag = + 0.88$ V

Based on the above potentials, strongest oxidizing agent will be :

(A) Cu^{+} (B) I_2 (C*) Ag^{+} (D) Fe^{3+}

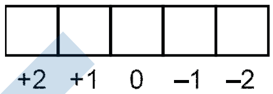
Sol. Higher the value of reduction potential stronger will be the oxidising hence based on the given values Ag^{+} will be strongest oxidizing agent.

60. In an atom how many orbital(s) will have the quantum numbers ; n = 3, ℓ = 2 and m_ℓ = + 2 ?

(A) 3 (B*) 1 (C) 5 (D) 7

Sol. n = 3, ℓ = 2 means 3d orbital

i.e. in an atom only one orbital can have the value m_ℓ = +2



PART-C-MATHEMATICS

61. A value of x for which $\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1}x)$, is :

(A) 1 (B) $\frac{1}{2}$ (C*) $\frac{-1}{2}$ (D) 0

Sol. $\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1} x)$

$\Rightarrow \text{cosec}^2(\cot^{-1}(1 + x)) = \sec^2(\tan^{-1} x)$

$\Rightarrow 1 + [\cot(\cot^{-1}(1 + x))]^2 = 1 + [\sec(\tan^{-1} x)]^2$

$\Rightarrow (1 + x)^2 = x^2 \Rightarrow x = -\frac{1}{2}$

62. If $z_1 \neq 0$ and z_2 be two complex numbers such that $\frac{z_2}{z_1}$ is a purely imaginary number, then $\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right|$ is equal to :

(A*) 1 (B) 3 (C) 5 (D) 2

Sol. Let $z_1 = 1 + I$ and $z_2 = 1 - i$

$$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$$

$$\frac{2z_1+3z_2}{2z_1-3z_2} = \frac{2+3\left(\frac{z_2}{z_1}\right)}{2-3\left(\frac{z_2}{z_1}\right)} = \frac{2-3i}{2+3i}$$

$$\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right| = \left|\frac{2-3i}{2+3i}\right| = \frac{|2-3i|}{|2+3i|}$$

$$\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1$$

63. A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes atleast 1 lady, atleast 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is :

(A) 40 (B*) 41 (C) 16 (D) 32

Sol.

L	O	Y
2	2	4
≥1	≥1	2≤

⇒

L	O	Y
1	1	2
1	2	1
2	1	1
2	2	0

Required number of ways

$= {}^2C_1 \times {}^2C_1 \times {}^2C_2 + {}^2C_1 \times {}^2C_2 \times {}^4C_1 + {}^2C_2 \times {}^2C_1 \times {}^4C_1 + {}^2C_2 \times {}^2C_2 \times {}^2C_0$

$= 2 \times 2 \times \frac{4 \times 3}{2} + 2 \times 1 \times 4 + 1 \times 2 \times 4 + 1 \times 1 \times 1$

$= 24 + 8 + 8 + 1 = 41$

64. Let $f(x) = \frac{x^2 - x}{x^2 + 2x}$, $x \neq 0, -2$. Then $\frac{d}{dx}\left[f^{-1}(x)\right]$ (wherever it is defined) is equal to :

(A*) $\frac{3}{(1-x)^2}$ (B) $\frac{-1}{(1-x)^2}$ (C) $\frac{-3}{(1-x)^2}$ (D) $\frac{1}{(1-x)^2}$

Sol. Let $y = \frac{x^2 - x}{x^2 + 2x}$

$\Rightarrow (x^2 + 2x)y = x^2 - x$

$\Rightarrow x(x + 2)y = x(x - 1)$

$\Rightarrow x[(x + 2)y - (x - 1)] = 0$

$\because x \neq 0, \therefore (x + 2)y - (x - 1) = 0$

$\Rightarrow xy + 2y - x + 1 = 0$

$\Rightarrow x(y-1) = -(2y+1)$

$\therefore\ x = \frac{2y+1}{1-y} \Rightarrow f^{-1}(x) = \frac{2x+1}{1-x}$

$\frac{d}{dx}(f^{-1}(x)) = \frac{2(1-x)-(2x+1)(-1)}{(1-x)^2}$

$= \frac{2-2x+2x+1}{(1-x)^2} = \frac{3}{(1-x)^2}$

65. Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is :

(A) $3y + 1 = 0$ (B) $y + 3 = 0$ (C*) $3y - 1 = 0$ (D) $y - 3 = 0$

Sol. $x^2 - 8y$ ...(i)

$\frac{x^2}{3} + y^2 = 1$ ...(ii)

From (i) and (ii)

$\frac{8y}{3} + y^2 = 1 \Rightarrow y - 3, \frac{1}{3}$

When $y = -3$, then $x^2 = -24$, which is not possible.

When $y = \frac{1}{3}$, then $x = \pm\frac{2\sqrt{6}}{3}$

Point of intersection are

$\left(\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$ and $\left(-\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$

Required equation of the line,

$y - \frac{1}{3} = 0 \Rightarrow 3y - 1 = 0$

66. If the three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ form a right-angled triangle then :

(A*) $a^2 - 9a + 18 = 0$ (B) $a^2 - 6a - 18 = 0$ (C) $a^2 - 9a + 12 = 0$ (D) $a^2 - 6a - 12 = 0$

Sol. Since three lines $x - 3y = p$,

$ax + 2y = q$ and $ax + y = r$

$\therefore$ product of slopes of any two lines = – 1

Suppose $ax + 2y = q$ and $x - 3y = p$ are $\perp$ to each other.

$\therefore \frac{-a}{2} \times \frac{1}{3} = -1 \Rightarrow a = 6$

Now, consider option are by one a = 6 satisfies only option (a)

$\therefore$ Required answer is $a^2 - 9a + 18 = 0$

67. Let A = {1, 2, 3, 4} and R : A → A be the relation defined by :

R = {(1, 1), (2, 3), (3, 4), (4, 2)}.

The correct statement is

(A*) R is an onto function.

(B) R does not have an inverse.

(C) R is not a one to one function.

(D) R is not a function.

Sol. Domain = {1, 2, 3, 4}

Range = {1, 2, 3, 4}

∴ Domain = Range

Hence the relation R is onto function.

68. A vector $\vec{n}$ is inclined to x-axis at 45°, to y-axis at 60° and at an acute angle to z-axis. If is a normal to a plane passing through the point ($\sqrt{2}$, – 1, 1), then the equation of the plane is :

(A) $3\sqrt{2}x - 4y - 3z = 7$

(B) $4\sqrt{2}x + 7y + z = 2$

(C*) $2x + y + 2z = 2\sqrt{2} + 1$

(D) $\sqrt{2}x - y - z = 2$

Sol. Direction cosines of $\vec{n}$ are $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$.

Equation of the plane,

$$\frac{1}{2}\left(x-\sqrt{2}\right)+\frac{1}{2}(y+1)+\frac{1}{2}(z-1)=0$$

$$\Rightarrow 2\left(x-\sqrt{2}\right)+\frac{1}{2}(y+1)+2(z-1)=0$$

$$\Rightarrow 2x+y+2z=2\sqrt{2}-1+2$$

$$\Rightarrow 2x+y+2z=2\sqrt{2}+1$$

69. If a and c are positive real numbers and the ellipse $\frac{x^2}{4c^2}+\frac{y^2}{c^2}=1$ has four distinct points in common with the circle $x^2 + y^2 = 9a^2$, then :

(A*) $9ac - 9a^2 - 2c^2 > 0$

(B) $6ac + 9a^2 - 2c^2 < 0$

(C) $6ac + 9a^2 - 2c^2 > 0$

(D) $9ac - 9a^2 - 2c^2 < 0$

Sol. Radius = 3a

length of major axis = 4c

Now, (Radius) < (Half of the length of major axis)

$3a < 2c$

$9a^2 < 4c^2$

$9ac - 9a^2 > 9ac - 4c^2$

$9ac - 9a^2 - 2c^2 > 9ac - 9c^2$ ...(i)

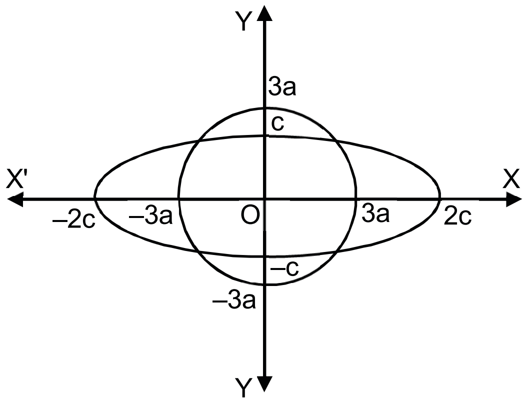
Again $3a < 2c$

$\Rightarrow 9ac < 6c^2$

$\Rightarrow 9ac - 6c^2 < 0$ ....(ii)

From (i) and (ii),

$9ac - 9a^2 - 2c^2 > 0$



70. **Statement-1:** The slope of the tangent at any point P on a parabola, whose axis is the axis of x and vertex is at the origin, is inversely proportional to the ordinate of the point P.

Statement-2: The system of parabolas $y^2 = 4ax$ satisfies a differential equation of degree 1 and order 1.

(A) Statement-1 is true; Statement-2 is false

(B) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(C*) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(D) Statement-1 is false; Statement-2 is true.

Sol. **Statement-1 :** $y^2 = \pm 4ax$

$\Rightarrow \frac{dy}{dx} = \pm 2a.\frac{1}{y} \Rightarrow \frac{dy}{dx} \propto \frac{1}{y}$

Statement-2: $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$

Thus both statements are true but statement 2 is not a correct explanation for statement-1

71. **Statement-1:** The equation x log x = 2 – x is satisfied by atleast one value of x lying between 1 and 2.

Statement-2: The function f(x) = x log x is an increasing function in [1, 2] and g(x) = 2 – x is a decreasing function in [1, 2] and the graphs represented by these functions intersect at a point in [1, 2]

(A) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(B) Statement-1 is true; Statement-2 is false

(C*) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

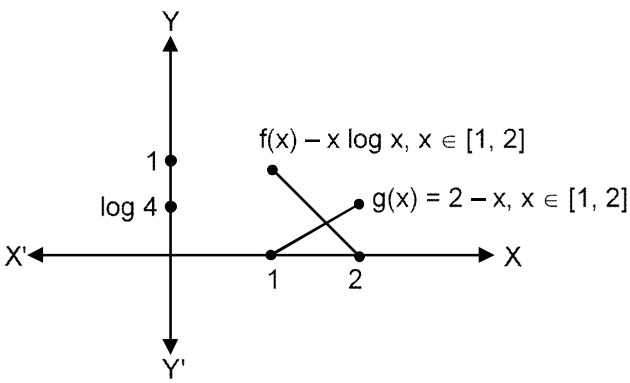
(D) Statement-1 is false; Statement-2 is true.

Sol. $f(x) = x \log x$, $f(1) = 0$, $f(2) = 4$

$g(x) = 2 - x$, $g(1) = 1$, $g(2) = 0$

$\log 10 > \log 4 \Rightarrow 1 > \log 4$

Thus statement-1 and 2 both are true and statement-2 is a correct explanation of statement 1.



72. If $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$, then $\frac{d^2y}{dx^2}$ is equal to :

(A) $\frac{y}{\sqrt{1+y^2}}$ (B*) y (C) $\sqrt{1+y^2}$ (D) y^2

Sol. $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$

$\Rightarrow 1 = \frac{1}{\sqrt{1+y^2}} \cdot \frac{dy}{dx}$

$\left[\because \text{ If } I(x) = \int_{\phi(x)}^{\psi(x)} f(t)dt, \text{ then} \frac{dI(x)}{dx} = f\{\psi(x)\}\right]$

$\left\{\frac{d}{dx}\psi(x)\right\} - f\{\phi(x)\}.\left\{\frac{d}{dx}\phi(x)\right\}$

$\frac{dy}{dx} = \sqrt{1-y^2}$

$\Rightarrow \frac{d^2y}{dx^2} \frac{1}{2\sqrt{1+y^2}}.2y\frac{dy}{dx} = \frac{y}{\sqrt{1+y^2}}\sqrt{1+y^2} = y$

73. If a, b, c are sides of a scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is :

(A) non-positive (B*) negative (C) positive (D) non-negative

Sol. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$

$= (a + b + c) [ab + bc + ca – a^2 – b^2 – c^2]$

$= –(a + b + c) [(a – b^2) + (b – c)^2 + (c – a)^2]$

Since a, b, c are sides of a scalene triangle, therefore at least two of the a, b, c will be unequal.

$\therefore$ $(a – b)^2 + (b – c)^2 + (c – a)^2 > 0$

Also $a + b + c > 0$

$\therefore$ $–(a + b + c) [(a– b)^2 + (b – c)^2 + (c – a)^2] < 0$

74. Let a_1, a_2, a_3 , ... be an A.P. such that $\dfrac{a_1 + a_2 + \ldots\ldots + a_p}{a_1 + a_2 + a_3 + \ldots\ldots + a_q} = \dfrac{p^3}{q^3}$; $p \neq q$.

Then $\dfrac{a_6}{a_{21}}$ is equal to :

(A) $\dfrac{11}{41}$ (B) $\dfrac{41}{11}$ (C) $\dfrac{121}{1861}$ (D*) $\dfrac{31}{121}$

Sol. $\dfrac{a_1 + a_2 + a_3 + \ldots\ldots + a_p}{a_1 + a_2 + a_3 + \ldots\ldots + a_q} = \dfrac{p^3}{q^3}$

$\Rightarrow \dfrac{a_1 + a_2}{a_1} = \dfrac{8}{1} \Rightarrow a_1 + (a_1 + d) = 8a_1$

$\Rightarrow$ d = $6a_1$

Now $\dfrac{a_6}{a_{21}} = \dfrac{a_1 + 5d}{a_1 + 20d}$

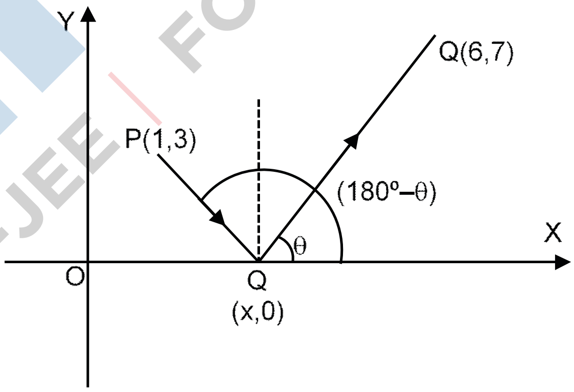
$\dfrac{a_1 + 5 \times 6a_1}{a_1 + 20 \times 6a_1} = \dfrac{1 + 30}{1 + 120} = \dfrac{31}{121}$

75. A light ray emerging from the point source placed at P (1, 3) is reflected at a point Q in the axis of x. If the reflected ray passes through the point R (6, 7), then the abscissa of Q is :

(A) 3 (B*) $\dfrac{5}{2}$ (C) $\dfrac{7}{2}$ (D) 1

Sol. Let abscissa of Q = x

$\therefore$ Q = (x, 0)



$\tan\theta = \dfrac{0-7}{x-6}, \tan(180° - \theta) = \dfrac{0-3}{x-1}$

Now, tan (180° – θ) = – tan θ

$\therefore \dfrac{-3}{x-1} = \dfrac{-7}{x-6} \Rightarrow x = \dfrac{5}{2}$

76. If $\int \dfrac{dx}{x + x^7} = p(x)$, then $\int \dfrac{x^6}{x + x^7} dx$ is equal to :

(A) ln | x | + p(x) + C (B) x + p(x) + C (C*) ln | x | – p(x) + C (D) x – p(x) + C

Sol. $\int \dfrac{x^6}{x + x^7} dx = \int \dfrac{x^6}{x(1 + x^6)} dx$

$= \int \dfrac{(1 + x^6) - 1}{x(1 + x^6)} dx$

$= \int \dfrac{1}{x} dx - \int \dfrac{1}{x + x^7} dx$

= ln | x | – p(x) + C

77. The matrix $A^2 + 4A - 5I$, where I is identity matrix and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, equals :

(A) $2\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ (B*) $2\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ (C) $32\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (D) $4\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

78. If the surface area of a sphere of radius r is increasing uniformly at the rate 8 cm^2/s, then the rate of change of its volume is :

(A) proportional to $\sqrt{r}$ (B) constant (C*) proportional to r (D) proportional to r^2

Sol. $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 . \frac{dr}{dt}$ ....(i)

$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r . \frac{dr}{dt}$

$\Rightarrow 8 = 8\pi r \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi r}$

Putting the value of $\frac{dr}{dt}$ in (i), we get

$\frac{dV}{dt} = 4\pi r^2 \times \frac{1}{\pi r} = 4r$

$\Rightarrow \frac{dV}{dt}$ is proportional to r.

79. The probability of a man hitting a target is $\frac{2}{5}$. He fires at the target k times (k, a given number). Then the minimum k, so that the probability of hitting the target at least once is more than $\frac{7}{10}$, is :

(A) 5 (B) 4 (C*) 3 (D) 2

Sol. $\frac{2}{5} + \frac{3}{5} \times \frac{2}{5} + \left(\frac{3}{5}\right)^2 \times \frac{2}{5} + ... + \left(\frac{3}{5}\right)^k . \frac{2}{5} > \frac{7}{10}$

$\Rightarrow \frac{2}{5}\left[1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + ... + \left(\frac{3}{5}\right)^k\right] > \frac{7}{10}$

$\Rightarrow \frac{2}{5} \times \frac{1 - \left(\frac{3}{5}\right)^k}{1 - \frac{3}{5}} > \frac{7}{10} \Rightarrow 1 - \left(\frac{3}{5}\right)^k > \frac{7}{10}$

$\Rightarrow \left(\frac{3}{5}\right)^k < \frac{3}{10} \Rightarrow k \geq 3$

Hence minimum value of k = 3

80. The vector $\left(\hat{i} \times \vec{a} \cdot \vec{b}\right)\hat{i} + \left(\hat{j} \times \vec{a} \cdot \vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a} \cdot \vec{b}\right)\hat{k}$ is equal to :

(A) $\vec{b}$ (B*) $\vec{a} \times \vec{b}$ (C) $\vec{a}$ (D) $\vec{b} \times \vec{a}$

Sol. $\left(\hat{i}\times\vec{a}\cdot\vec{b}\right)\hat{i}+\left(\hat{j}\times\vec{a}\cdot\vec{b}\right)\hat{j}+\left(\hat{k}\times\vec{a}\cdot\vec{b}\right)\hat{k}$

$=\left(\hat{i}\cdot\vec{a}\times\vec{b}\right)\hat{i}+\left(\hat{j}\cdot\vec{a}\times\vec{b}\right)\hat{j}+\left(\hat{k}\cdot\vec{a}\times\vec{b}\right)\hat{k}$

$\left(\because\ \vec{a}\times\vec{b}\cdot\vec{c}=\vec{a}\cdot\vec{b}\times\vec{c}\right)$

$=\left(\vec{a}\times\vec{b}\right)\hat{i}+\left(\vec{a}\times\vec{b}\right)\hat{j}+\left(\vec{a}\times\vec{b}\right)\hat{k}$

$=\vec{a}\times\vec{b}$

81. The area bounded by the curve y = ln (x) and the lines y = 0, y = ln (C) and x = 0 is equal to :

(A) 3 (B) 3ln (C) – 2 (C*) 2 (D) 3ln (C) + 2

Sol. To find the point of intersection of curves

y = ln (x) and y = ln (3), put ln (x) = n (3)

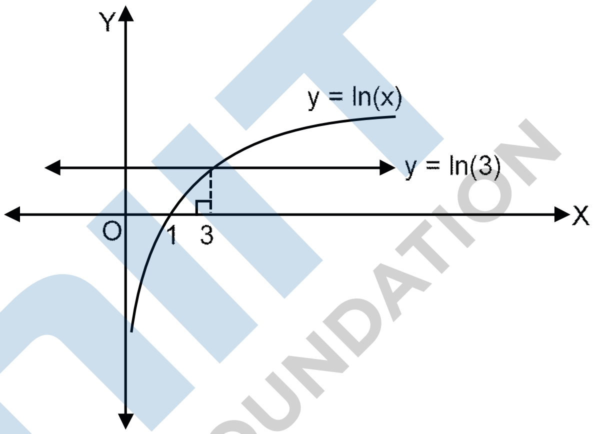
$\Rightarrow$ ln (x) – ln (3) = 0

$\Rightarrow$ ln (x) – ln (3) = ln (1)

$\Rightarrow \frac{x}{3}=1, \Rightarrow x = 3$

Required area $\int_0^3 \ln(3)dx-\int_1^3 \ln(x)dx$

$=[x\ln(3)]_0^3-[x\ln(x)-x_1^3]=2$



82. If the lines $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z+1}{3}$ and $\frac{x+2}{2}=\frac{y-k}{3}=\frac{z}{4}$ are coplanar, then the value of k is :

(A) $\frac{-9}{2}$ (B) $\frac{9}{2}$ (C*) $\frac{11}{2}$ (D) $\frac{-11}{2}$

Sol. $\begin{vmatrix}-2-(-1) & k-1 & 0-(-1)\\ 2 & 1 & 3\\ 2 & 3 & 4\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}-1 & k-1 & 1\\ 2 & 1 & 3\\ 2 & 3 & 4\end{vmatrix}=0$

$\Rightarrow$ (–1) (4 – 9) – (k – 1) (8 – 6) + 6 – 2 = 0

$\Rightarrow k=\frac{11}{2}$

83. If each of the lines 5x + 8y = 13 and 4x – y = 3 contains a diameter of the circle $x^2 + y^2 - 2(a^2 - 7a + 11)x - 2(a^2 - 6a + 6)y + b^3 + 1 = 0$, then :

(A) a = 2 and b $\in$ (– ∞, 1) (B) a = 5 and b $\notin$ (– 1, 1)

(C*) a = 5 and b $\in$ (– ∞, 1) (D) a = 1 and b $\notin$ (– 1, 1)

Sol. Point of intersection of two given lines is (1, 1). Since each of the two given lines contains a diameter of the given circle, therefore the point of intersection of the two given lines is the centre of the given circle.

Hence centre = (1, 1)

$\therefore\ a^2 - 7a + 11 = 1 \Rightarrow a = 2, 5$ ....(i)

and $a^2 - 6a + 6 = 1 \Rightarrow a = 1, 5$ ....(ii)

From both (i) and (ii), a = 5

Now on replacing each of $(a^2 - 7a + 11)$ and $(a^2 - 6a + 6)$ by 1, the equation of the given circle $x^2 + y^2 - 2x - 2y + b^3 + 1 = 0$

$\Rightarrow (x - 1)^2 + (y - 1)^2 + b^3 = 1$

$\Rightarrow b^3 = 1 - [(x - 1)^2 + b^3 = 1]$

$\therefore\ b \in (-\infty, 1)$

84. The values of 'a' for which one root of the equation $x^2 - (a + 1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2 are given by :

(A) $3 < a < 10$ (B*) $-2 < a < 3$ (C) $a \le -2$ (D) $a \ge 10$

Sol. $x - (a + 1)x + a^2 + a - 8 = 0$

Since roots are different, therefore D > 0

$\Rightarrow (a + 1)^2 - 4(a^2 + a - 8) > 0$

$\Rightarrow (a - 3)(3a + 1) < 0$

There are two cases arises.

Case I. $a - 3 > 0$ and $3a + 1 < 0$

$\Rightarrow a > 3$ and $a < -\frac{11}{3}$

Hence, no solution in this case

Case II: $a - 3 < 0$ and $3a + 11 > 0$

$\Rightarrow a < 3$ and $a > -\frac{11}{3}$

$\therefore -\frac{11}{3} < a < 3 \Rightarrow -2 < a < 3$

85. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector of the type $\vec{b} + \lambda\vec{c}$ for some scalar λ, whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is :

(A*) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (B) $2\hat{i} - \hat{j} + 5\hat{k}$ (C) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

Sol. Let $\vec{d} = \vec{b} + \lambda\vec{c}$

$\therefore \vec{d} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

$= (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}$

If θ be the angle between $\vec{d}$ and $\vec{a}$, then projection of $\vec{d}$ or $(\vec{b} + \lambda\vec{c})$ on $\vec{a} = |\vec{d}|\cos\theta$

$=|\vec{d}|\left(\frac{\vec{d}\cdot\vec{a}}{|\vec{d}||\vec{a}|}\right)=\frac{\vec{d}\cdot\vec{a}}{|\vec{a}|}$

$=\frac{2(\lambda+1)-(\lambda+2)-(2\lambda+1)}{\sqrt{4+1+1}}$

$=\frac{-\lambda-1}{\sqrt{6}}$

But projection of $\vec{d}$ on $\vec{a}=\sqrt{\frac{2}{3}}$

$\therefore -\frac{\lambda+1}{\sqrt{6}}=\sqrt{\frac{2}{3}} \Rightarrow \frac{\lambda^2+2\lambda+1}{6}=\frac{2}{3}$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda + 3) - 1 (\lambda + 3) = 0, \Rightarrow \lambda = 1, -3$

when $\lambda = 1$, $\vec{b}+\lambda\vec{c}=2\hat{i}+3\hat{j}-3\hat{k}$

when $\lambda = -3$, then $\vec{b}+\lambda\vec{c}=-2\hat{i}-\hat{j}+5\hat{k}$

86. **Statement-1:** The statement $A \to (B \to A)$ is equivalent to $A \to (A \vee B)$.

Statement-2: The statement $\sim [(A \wedge B) \to (\sim A \vee B)]$ is a Tautology.

(A*) Statement-1 is true; Statement-2 is false

(B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(C) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(D) Statement-1 is false; Statement-2 is true.

Sol.

A	B	~ A	$A \wedge B$	$\sim A \vee B$	$(A \wedge B) \to (\sim A \vee B)$	$\sim [(A \wedge B) \to (\sim A \vee B)]$
T	T	F	T	T	T	F
T	F	F	F	F	T	F
F	T	T	F	T	T	F
F	F	T	F	T	T	F

87. The mean of a data set consisting of 20 observations is 40. If one observation 53 was wrongly recorded as 33, then the correct mean will be :

(A) 49 (B) 40.5 (C) 42.5 (D*) 41

Sol. Correct mean $=\frac{20\times 40-33+55}{20}=41.1$

Nearest option : 41

88. The value of $\lim_{x\to 0}\frac{1}{x}\left[\tan^{-1}\left(\frac{x+1}{2x+1}\right)-\frac{\pi}{4}\right]$ is :

(A*) $\frac{-1}{2}$ (B) 1 (C) 2 (D) 0

Sol. $\lim_{x\to 0}\left(\frac{1}{x}\right)\left[\tan^{-1}\left(\frac{x+1}{2x+1}\right)-\frac{\pi}{4}\right]$

$$= \lim_{x\to 0}\left(\frac{1}{x}\right)\left[\tan^{-1}\left(\frac{x+1}{2x+1}\right) - \tan^{-1}(1)\right]$$

$$= \lim_{x\to 0}\left(\frac{1}{x}\right).\tan^{-1}\left(\frac{\frac{x+1}{2x+1}-1}{1+\frac{x+1}{2x+1}}\right)$$

$$= \lim_{x\to 0}\frac{1}{x}.\tan^{-1}\left(\frac{-x}{3x+2}\right)$$

$$= \lim_{x\to 0}\left[\frac{\tan^{-1}\left(\frac{x}{3x+2}\right)}{\frac{x}{3x+2}} \times \frac{1}{3x+2}\right] = -\frac{1}{2}$$

89. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2+\frac{2}{x}\right)^{15}$ is :

(A) 1 : 4 (B*) 1 : 32 (C) 7 : 64 (D) 7 : 16

Sol. $T_{r+1} = {}^{15}C_r(x^2)^{15-r} . (2x^{-1})^r = {}^{15}C_r \times (2)^r \times x^{30-3r}$

For independent term, $30 - 3r = 0 \Rightarrow r = 10$

Hence the term independent of x,

$T_{11} = {}^{15}C_{10} \times (2)^{10}$

For item involve x^{15}, $30 - 3r = 15 \Rightarrow r = 5$

Hence coefficient of $x^{15} = {}^{15}C_5 \times (2)^5$

Required ratio

$$= \frac{{}^{15}C_5 \times (2)^5}{{}^{15}C_{10} \times (2)^{10}} = \frac{\frac{15!}{10!5!}}{\frac{15!}{5!10!} \times (2)^5}$$

= 1 : 32

90. The sum of the series $1+\frac{1}{1+2}+\frac{1}{1+2+3}+......$ upto 10 terms, is :

(A) $\frac{22}{13}$ (B) $\frac{18}{11}$ (C) $\frac{16}{9}$ (D*) $\frac{20}{11}$

Sol. $T_r = \frac{1}{1+2+3+....+r} = \frac{2}{r(r+1)}$

$$S_{10} = \sum_{r=1}^{10}\frac{1}{r(r+1)} = 2\sum_{r=1}^{10}\left[\frac{r+1}{r(r+1)} - \frac{r}{r(r+1)}\right] = 2\sum_{r=1}^{10}\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$$= 2\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+.....+\left(\frac{1}{10}-\frac{1}{11}\right)\right]$$

$$= 2\left[1-\frac{1}{11}\right] = 2\times\frac{10}{11} = \frac{20}{11}$$